Solution sketch 1 - Computational Models - Spring 2013

1. Clues:
   (f) First show (using induction) that $\epsilon \cup 0(00 \cup 11)^*(0 \cup \epsilon)$ is a regular expression for the complement of the language
   (i) Note that a regular expression for the language is $(0 \cup 1)^*000(0 \cup 1)^*$

2. DFA: $A = (Q, \Sigma, \delta, q_0, F)$. $Q = \{x_1x_2 \cdots x_n : \forall i.x_i \in \{0,1\}\}$, $\Sigma = \{0,1\}$, $q_0 = 1^n$, $F = \{x_1x_2 \cdots x_n : x_1 = 0\}$, $\forall q \in Q, \sigma \in \Sigma, \delta(q, \sigma) = \delta(x_1x_2 \cdots x_n, \sigma) = x_2 \cdots x_n\sigma$.

3. This language is $L\bar{L} \cup \bar{L}L$. Since L is regular and the regular languages are closed under the operators complement, union and concatenation, the language at hand is also regular.

4. $\forall w \in \Sigma^*, \#_{01}(w) - \#_{10}(w) \in \{-1, 0, 1\}$ (why?). Therefore, we need only finite memory to keep track of that and decide this language. In other words, this language is $L(0(0 \cup 1)^*0 \cup 1(0 \cup 1)^*1)$ and is therefore regular.