Exercise 5 - Computational Models - Spring 2013

1. Prove $\{\langle M\rangle \mid L(M) = \Sigma^*\} \notin RE \cup CoRE$.

2. Describe an algorithms that decids $\{\langle M\ , w\rangle \mid M$ is a PDA that accepts $w\}$. Note that merely simulating a PDA using a NTM will not work - explain why.

3. (a) Show that $\overline{A_{TM}}$ reduces to A_{TM}

   (b) $\forall A, B$, If A reduces to B then $A \notin \mathcal{RE} \rightarrow \mathcal{B} \notin \mathcal{RE}$. True or False? Explain!

   (c) $\overline{A_{TM}} \leq_M A_{TM}$. True or False? Explain!

4. For the following decision problems determine whether they belong to $\mathcal{R}$, $\mathcal{RE} \setminus \mathcal{R}$, co-$\mathcal{RE} \setminus \mathcal{R}$ or none of the above:

   (a) Input: Turing machine M
   Question: is there an x for which M halts?

   (b) Input: Turing machine M
   Question: is there life beyond earth ?

   (c) Input: Turing machine M and inputs x and y
   Question: does M halt on exactly one of the inputs?

   (d) Input: Turing machine M
   Question: is $|L(M)| < 3$?

   (e) Input: Turing machine M
   Question: is $L(M) \in \mathcal{RE}$?

   (f) Input: Turing machine M such that $|\langle M\rangle| < 10^{100}$
   Question: does M halt on ϵ?

   (g) Input: Turing machine M
   Question: Is it true that for all inputs x, M's run over x never reaches position $|x| + 7$ on the tape?

(h) Input: Turing machine M whose transition function is in $Q \times \Gamma \rightarrow Q \times \Gamma \times \{R\}$ (i.e. TM that can only move right).
Question: Does M halt when it runs on ϵ ?

(i) Input: Turing machine M and a number k
Question: Does M halt on all inputs of length at most k^2 in k^3 steps?

(j) Input: LBAs A_1,A_2
Question: Does $L(A_1) = L(A_2)$?

(k) Input: CFG G
Question: Does $L(G) = 1(0 \cup 1)^*$?

5. Let S_n be the set of TMs with n states that halt on input ϵ. Denote by $M()$ the number of steps taken by M running on input ϵ. Define $f(m) = \min\{n \mid M() \geq m \text{ and } M \in S_n\}$.

   (a) Show that f is a total function from $\mathcal{N}$ to $\mathcal{N}$.

   (b) Show that f is monotone non decreasing.

   (c) Use BB(n) as defined in class to prove that f is not computable.