Computational Models, Spring 2013 Exrecise #4

Turing Machines $\mathcal{R}, \mathcal{RE}$, co-$\mathcal{RE}$

1. Let $A = \{Q_A, \Sigma, \delta_A, q_{0A}, F_A\}$ be a DFA and let $P = \{Q_P, \Sigma, \Gamma, \delta_P, q_{0P}, F_P\}$ be a PDA
   (a) Construct **formally** a TM that accepts $L(A)$
   (b) Construct **formally** a TM that accepts $L(P)$
2. Prove or disprove:
   (a) $\mathcal{R}$ is closed under complementation.
   (b) $\mathcal{RE}$ is closed under complementation.
   (c) $\mathcal{RE}$ is closed under intersection.
   (d) co-$\mathcal{RE}$ is closed under intersection.
   (e) $\mathcal{RE}$ is closed under Kleene star.
3. For each of the following languages, show that they are in $\mathcal{R}$.
   (a) $L = \{w\#w : w \in \{0,1\}^*\}$
   (b) $L = \{ww : w \in \{0,1\}^*\}$
4. Let L be a infinite language, prove or disprove:
   (a) For every such L there exists a language $L' \subset L$ such that L' **is not** decidable (i.e. $L' \notin \mathcal{R}$)
   (b) For every such L there exists a language $L' \subset L$ such that L' **is** decidable (i.e. $L' \in \mathcal{R}$)
5. Let $L_1, L_2 \in \mathcal{RE} \setminus \mathcal{R}$. Prove whether the following is possible:
   (a) $L_1 \cup L_2 \in \mathcal{R}$.
   (b) $L_1 \cup L_2 \in \mathcal{R}$ and $L_1 \cap L_2 \in \mathcal{R}$.