# HW 3 - Computational Models - Spring 2013

**Notation:** We denote by $\#_\sigma(w)$ the number of times the word $\sigma \in \Sigma^*$ is a substring in the word $w \in \Sigma^*$. We use the definition from class of a binary operation on languages over the same alphabet $\Sigma$:

$$L_1/L_2 = \{x \in \Sigma^* \mid \exists y \in L_2.\ \ xy \in L_1\}$$

1. (a) True or False ? (Prove your answer): $xy \in L_1 \circ L_2$ and $y \in L_2 \Rightarrow x \in L_1$
   (b) True or False ? (Prove your answer): $(L_1 \circ L_2)/L_2 = L_1$
   (c) What is $0^* \circ \{0^n 1^m \mid n < m \text{ and } n, m \in \mathbb{N}\}$?
   (d) What is $(01)^*/(0 \cup 1)^*0$?
   (e) What is $\{0^n 1^n \mid n \in \mathbb{N}\}/1^*$?
   (f) Give an example of a context free language $L_1$ and a language $L_2$ such that $L_1/L_2$ is not a context free language.

2. Determine whether the following languages are context free. Prove your answer.
   (a) $L_1 = \{a^{2n}b^{3n} | n \in \mathbb{N}\}$ over $\Sigma = \{a, b\}$.
   (b) $L_2 = \{0^n 1^{n^2} | n \in \mathbb{N}\}$ over $\Sigma = \{0, 1\}$.
   (c) $L_3 = \{x\#y | x, y \in \{0,1\}^* \wedge x \textit{ is a substring of } y\}$ over $\Sigma = \{0, 1, \#\}$.
   (d) $L_4 = \{w | w \neq w^R\}$ , over $\Sigma = \{0, 1\}$.

3. (a) Given a DFA $A$ with $n$ states:
   Prove that $|L(A)| = \infty \iff \exists w \in L(A)$ such that $n < |w| \leq 2n$
   (b) Use (a) and describe an algorithm that Given a DFA $A$ decides if $|L(A)| = \infty$
   (c) Given a CFG $G$ in Chomsky Normal Form with $n$ variables:
   Prove that $|L(G)| = \infty \iff \exists w \in L(G)$ such that $2^n < |w| \leq 2^{n+1}$
   (d) Use (c) and describe an algorithm that Given a CFG $G$ decides if $|L(G)| = \infty$