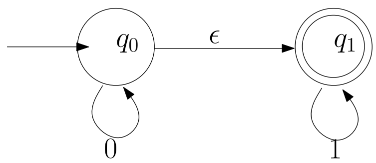
Computational Models, Spring 2013 Exrecise #2

DFA's, NFA's and non-regular languages

1. For each of the following languages draw an NFA, convert it to a DFA and draw the converted DFA.
   (a) $L = \{w : w \text{ contains the substring } 0011\}$ above $\Sigma = \{0, 1\}$
   (b) $L = \{w : \#_0(w) = 2 \text{ or } \#_1(w) = 1\}$ above $\Sigma = \{0, 1\}$
   (c) $L = \{a^i b^j c : i, j \geq 1\} \cup c^*$ above $\Sigma = \{a, b, c\}$
2. Write a regular expression for each of the following languages above. Give a thorough explanation (a formal proof is not needed)
   (a) $L = \{w : \ |w| \text{mod} 3 = 1\}$ above $\Sigma = \{a, b, c\}$
   (b) $L = \{w \in L[a^*b^*] \ : \ |w| \text{mod} 3 = 2\}$ above $\Sigma = \{a, b\}$
   (c) $L = \{w_1 w_2 : w_1 \in \{0^*\} \text{ and } w_2 \in \{101\}^* \text{ and } |w| \text{mod} 2 = 0\}$ above $\Sigma = \{0, 1\}$
3. Consider the following NFA



   (a) What is the regular expression that expresses the language?
   (b) Construct the minimal DFA representing the language
4. We showed in class that Regular languages are closed under union. Are they closed under infinite union? Prove your answer formally.
5. Let L be a regular language state for each language if it is regular or not. Give a complete formal proof:
   (a) $L_a = \{ww' : w \in L \text{ and } w' \notin L\}$
   (b) $L_b = \{y : \exists x, z \ s.t. |x| = |z| \text{ and } xyz \in L\}$
   (c) $L_c = \{xy : |x| = |y| \text{ and } \exists \sigma \ s.t. \ x\sigma y \in L\}$
   (d) $\text{Pref}(L) = \{x : \exists y \ s.t. \ xy \in L\}$
6. Prove that the following languages are not regular
   (a) $L_a = \{(ab)^n c^n : n \geq 0\}$ above $\Sigma = \{a, b, c\}$
   (b) $L_b = \{a^i b^j c^k : i = j \text{ or } j = k\}$ above $\Sigma = \{a, b, c\}$
   (c) $L_c = \{ww : w \in \{0, 1\}^*\}$
   (d) $L_d = \{\text{balanced strings }\}$ above $\Sigma = \{(,)\}$
   A string is balanced if the number of (equals the number of) and while reading the string, the number of (is equal or greater than the number of).