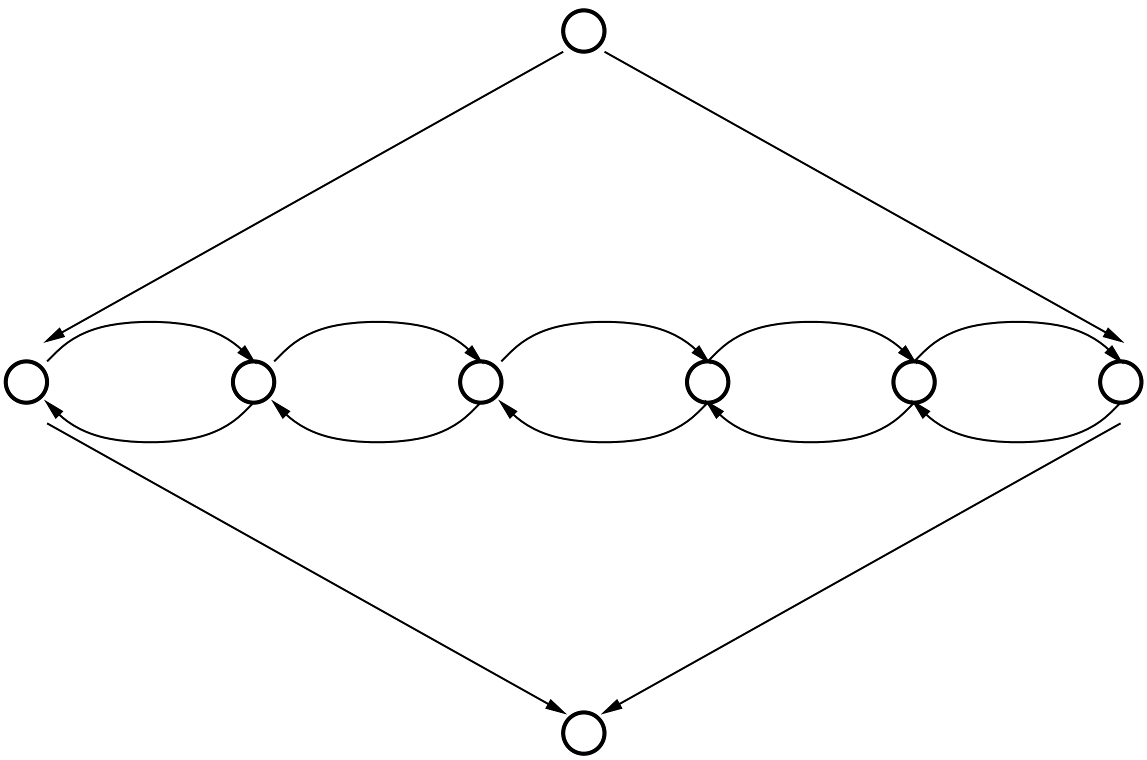
Hamiltonian Path

Each variable is represented by the following graph:

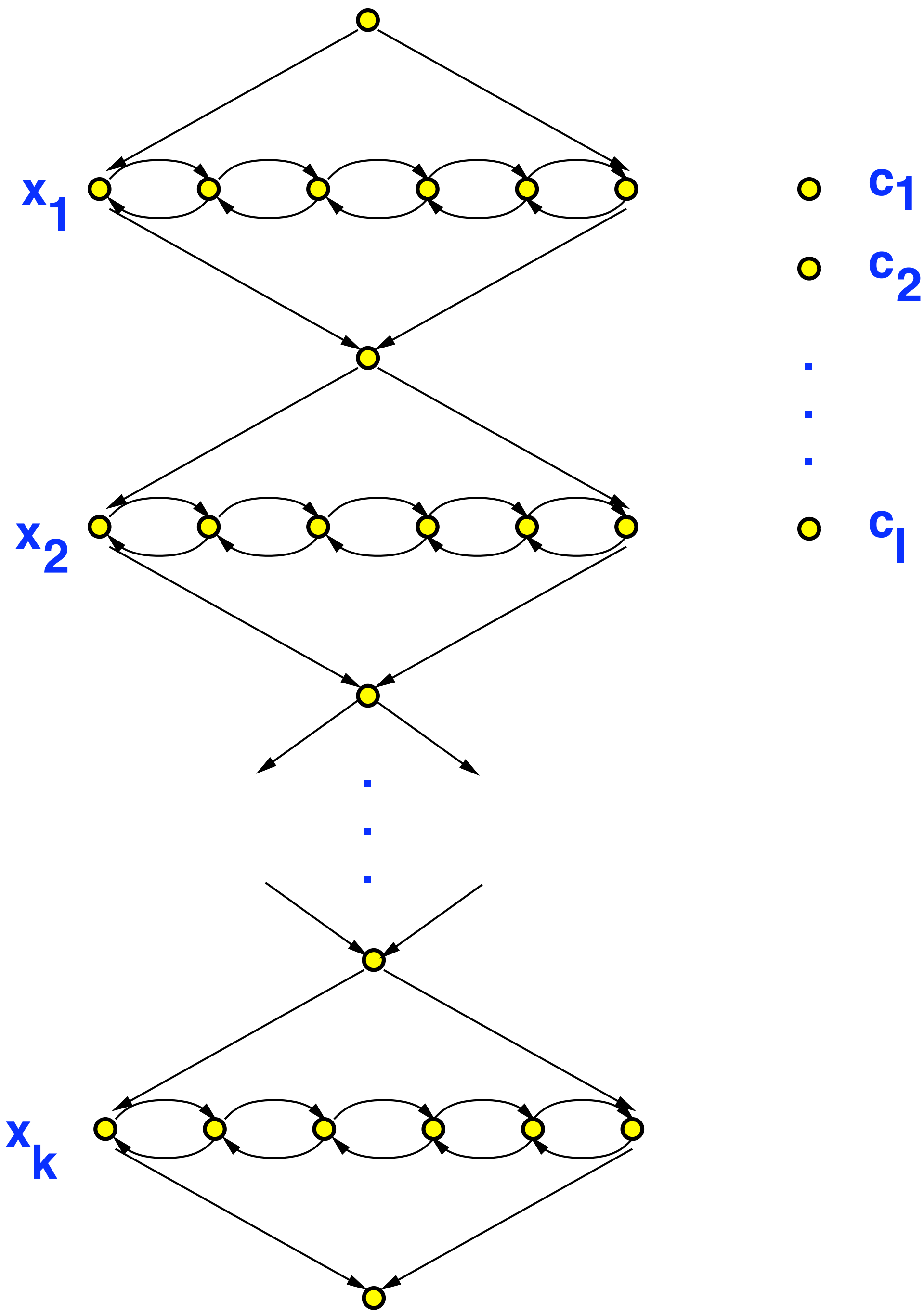


Hamiltonian Path

Each clause of ϕ is a single node.

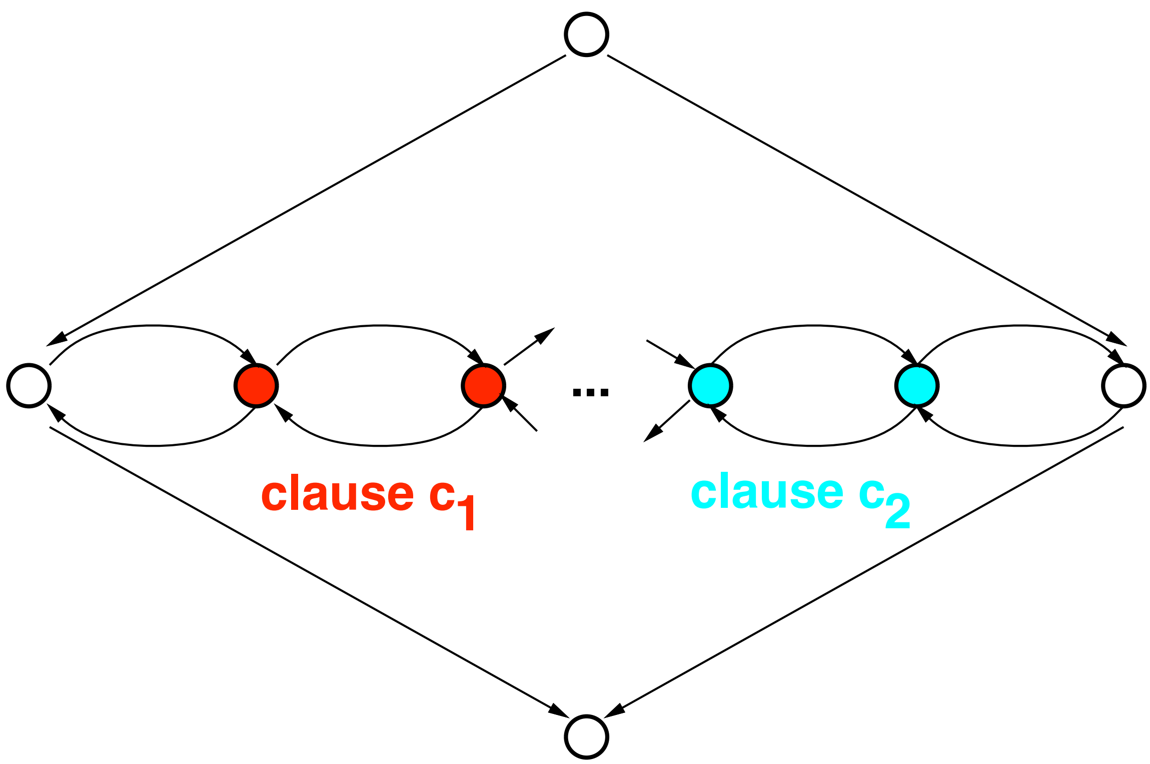
Hamiltonian Path

Global structure of graph (missing edges)



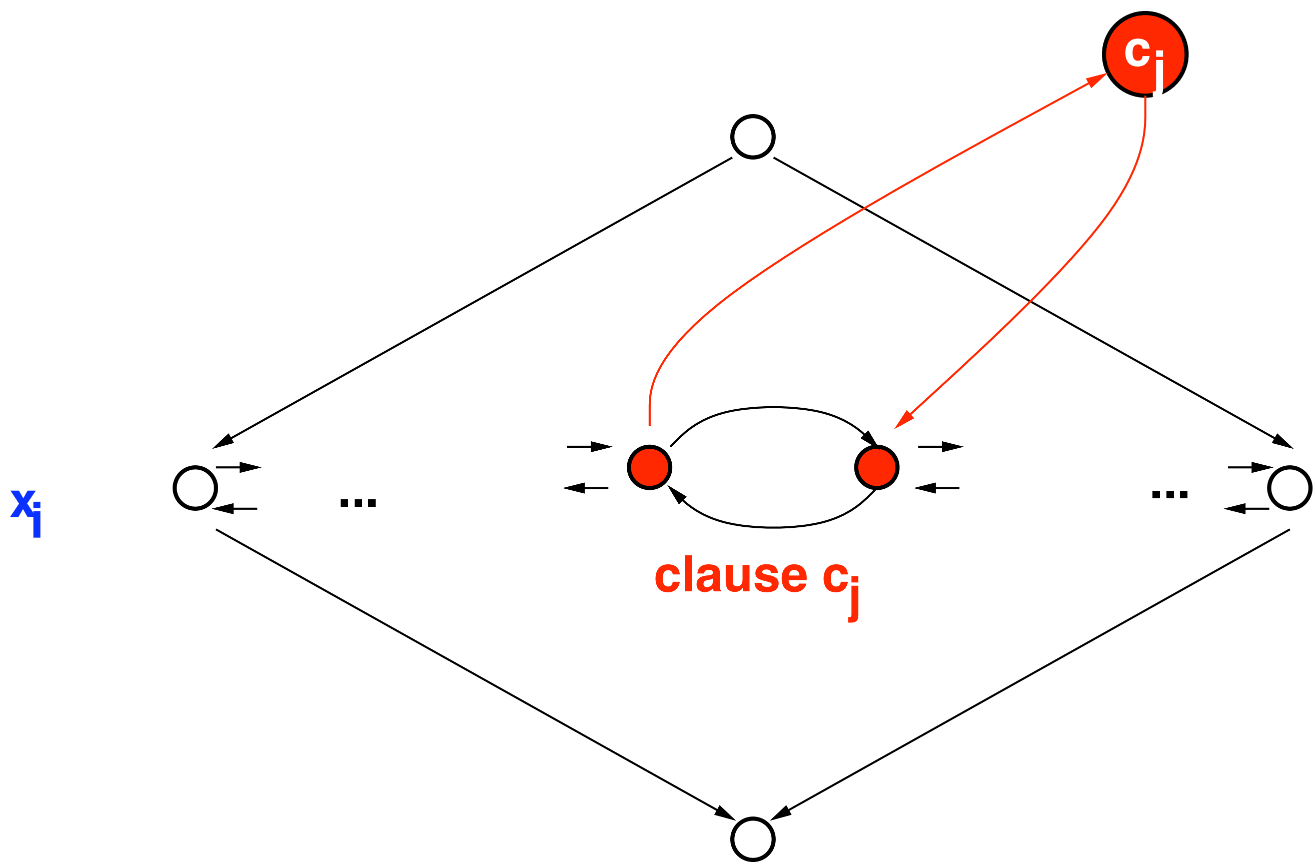
Hamiltonian Path

Center of each diamond has l pairs of nodes, one for each clause.



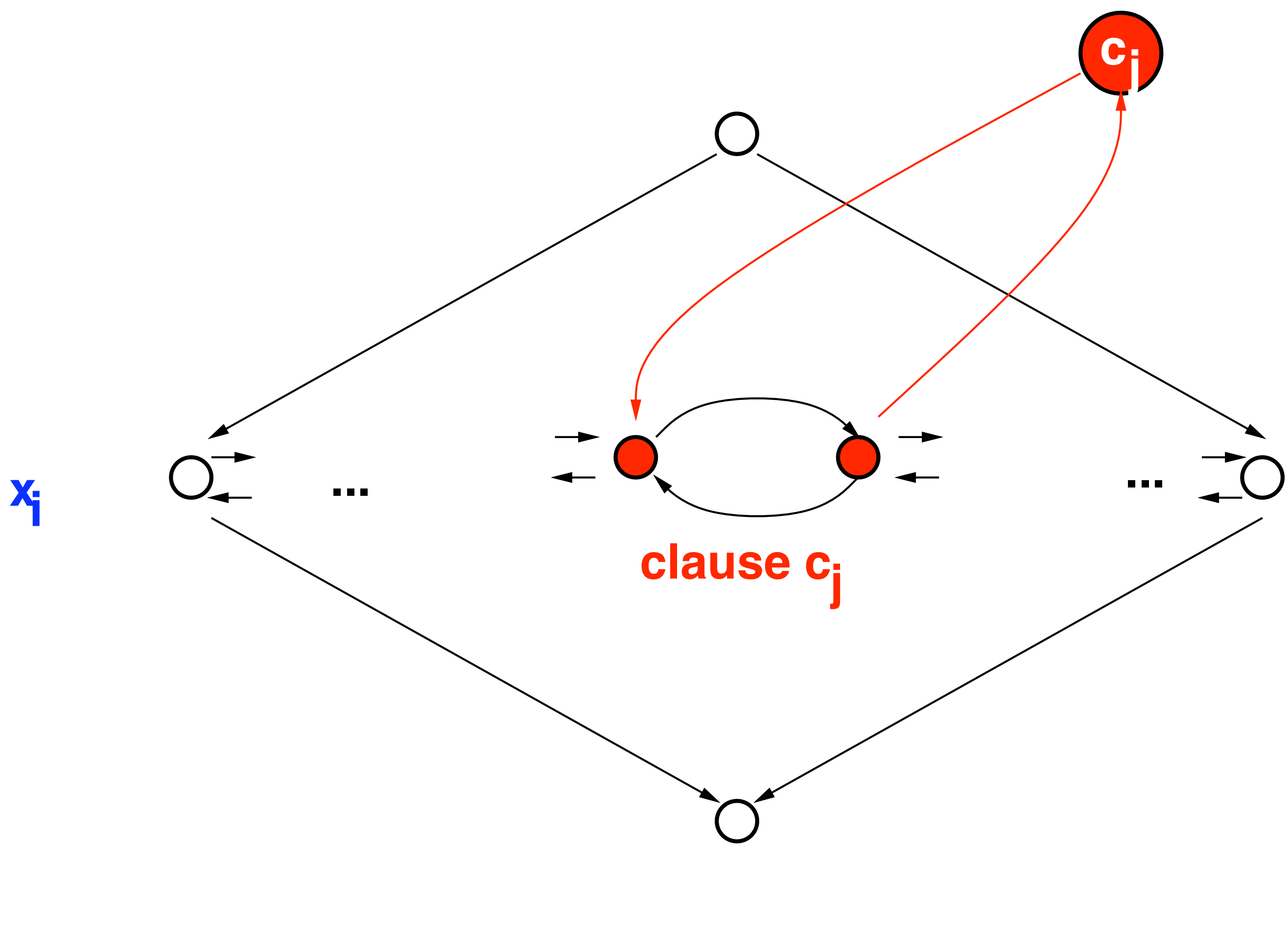
Hamiltonian Path

If variable x_i appears in clause c_j, add this "detour"



Hamiltonian Path

If $\overline{x_i}$ appears in clause c_j, add this "detour"



Hamiltonian Path

After adding edges from “diamonds” to clause vertexes, G is complete.

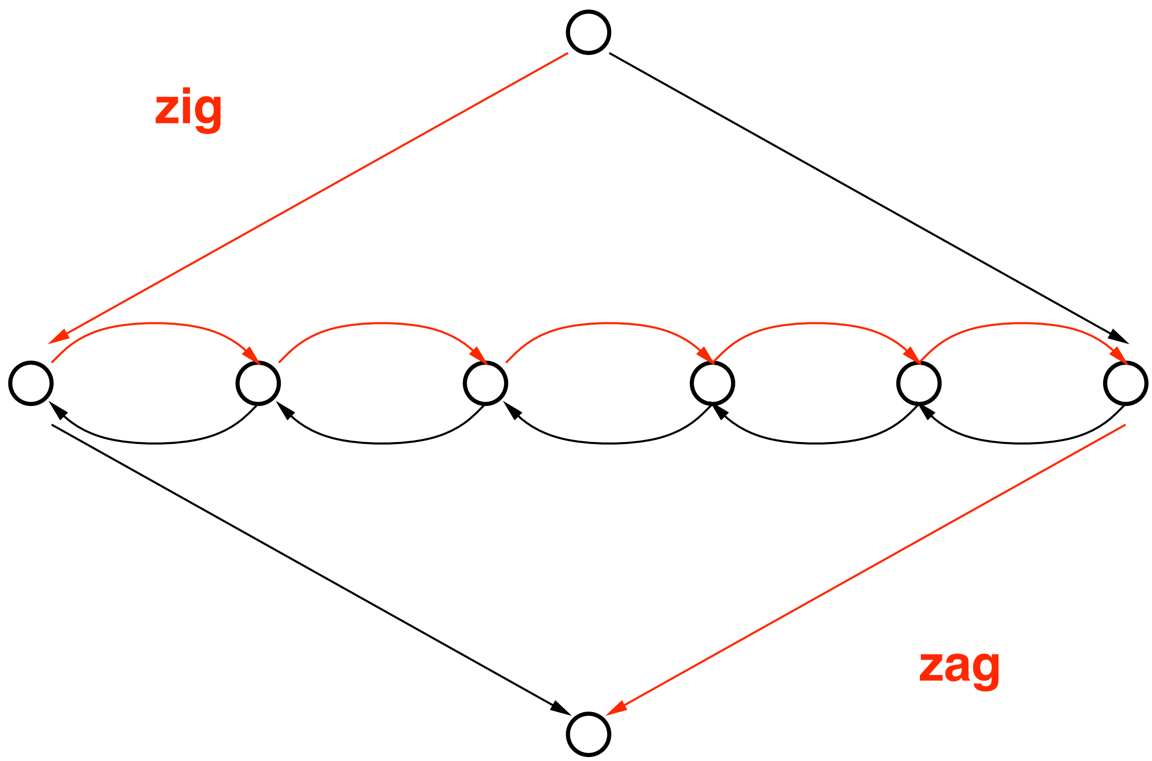
Claim: If ϕ is satisfiable, then G has a hamiltonian path.

Strategy:

- ignore clause nodes for now
- traverse diamonds

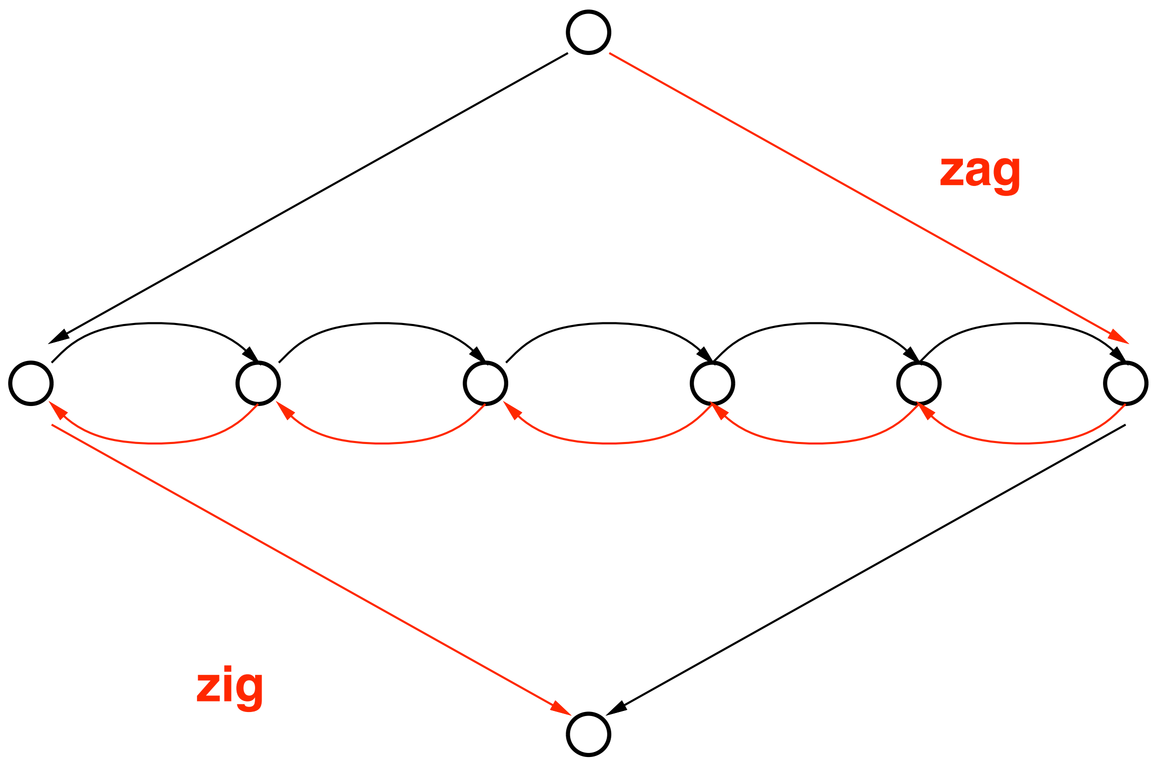
Hamiltonian Path

If x_i is **true** in the assignment, then **zig-zag**.



Hamiltonian Path

If x_i is **false** in the assignment, then **zag-zig**.

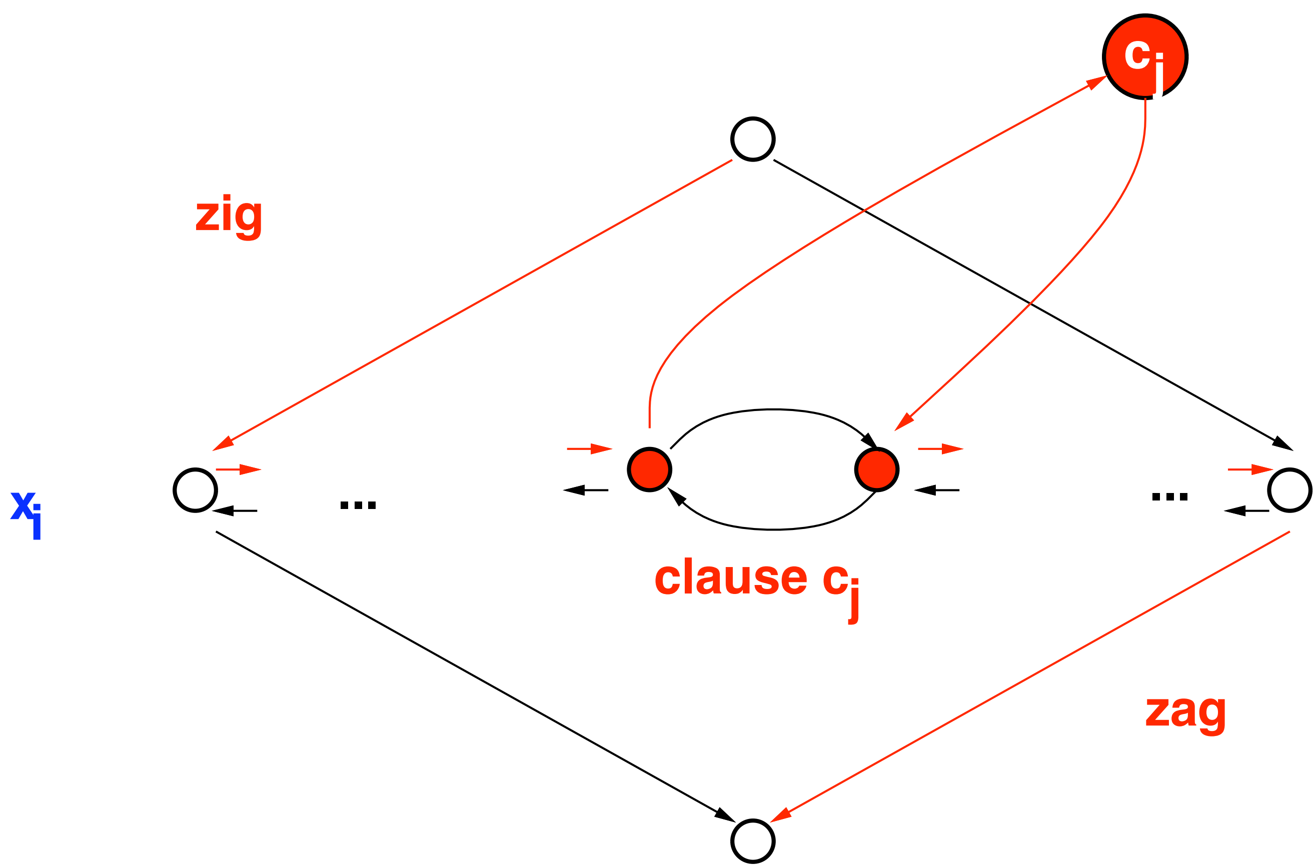


Hamiltonian Path

Add clause nodes.

- Each c_j is assigned one **true** literal.
- For each clause, pick one.

If we select x_i in c_i, add "detour"

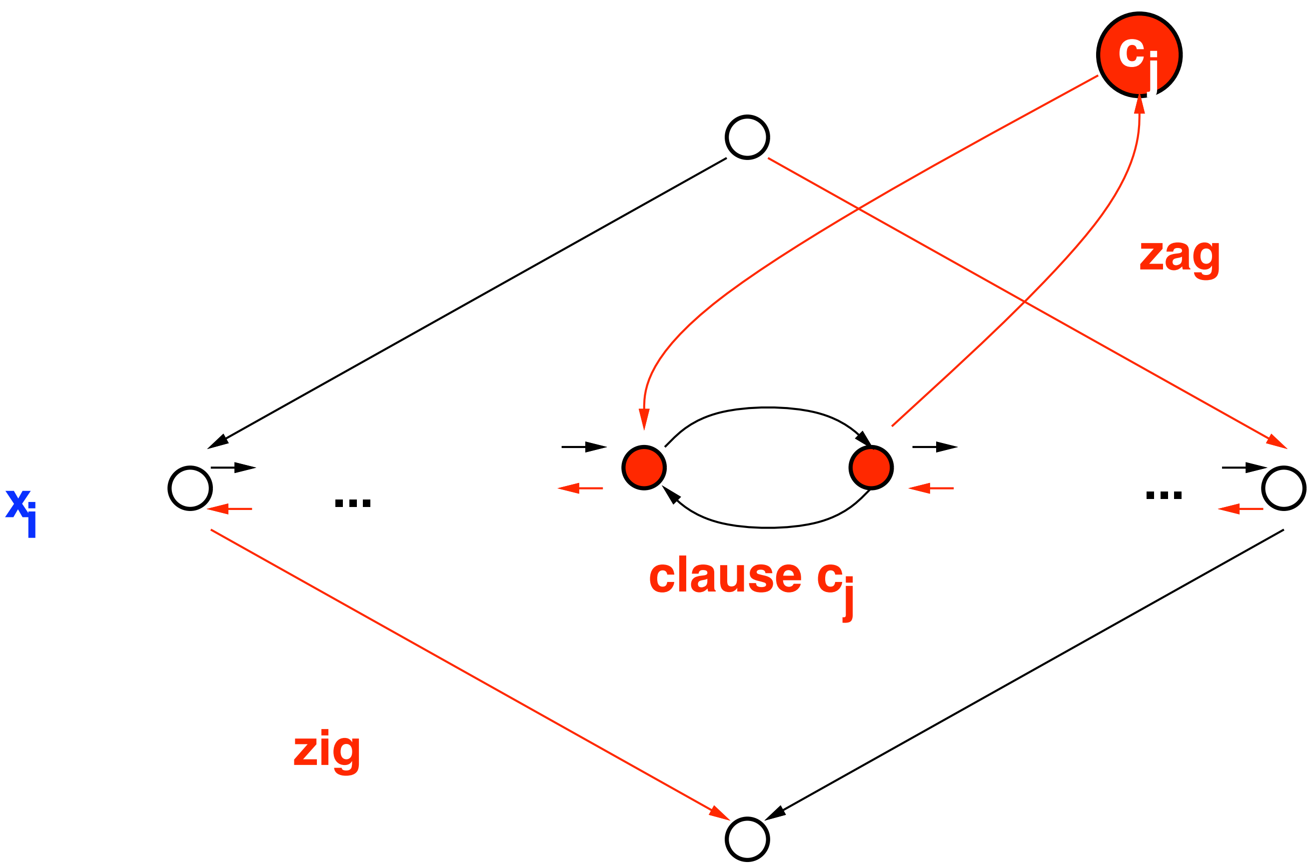


Hamiltonian Path

Add clause nodes.

- Each c_j is assigned one **true** literal.
- For each clause, pick one.

If we select $\overline{x_i}$ in c_i, add "detour"



This completes one direction of the reduction.

Hamiltonian Path

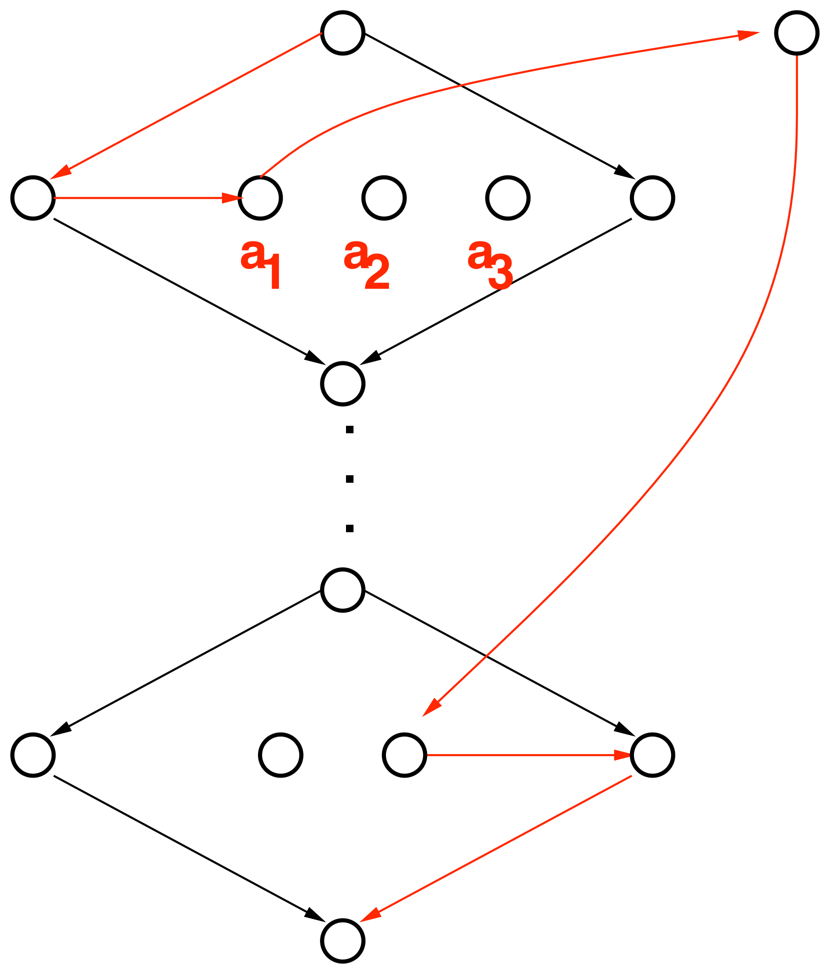
Claim: If G has a hamiltonian path from s to t, then ϕ has a satisfying assignment.

Definition: A **normal** hamiltonian path is one that traverses the diamonds in order.

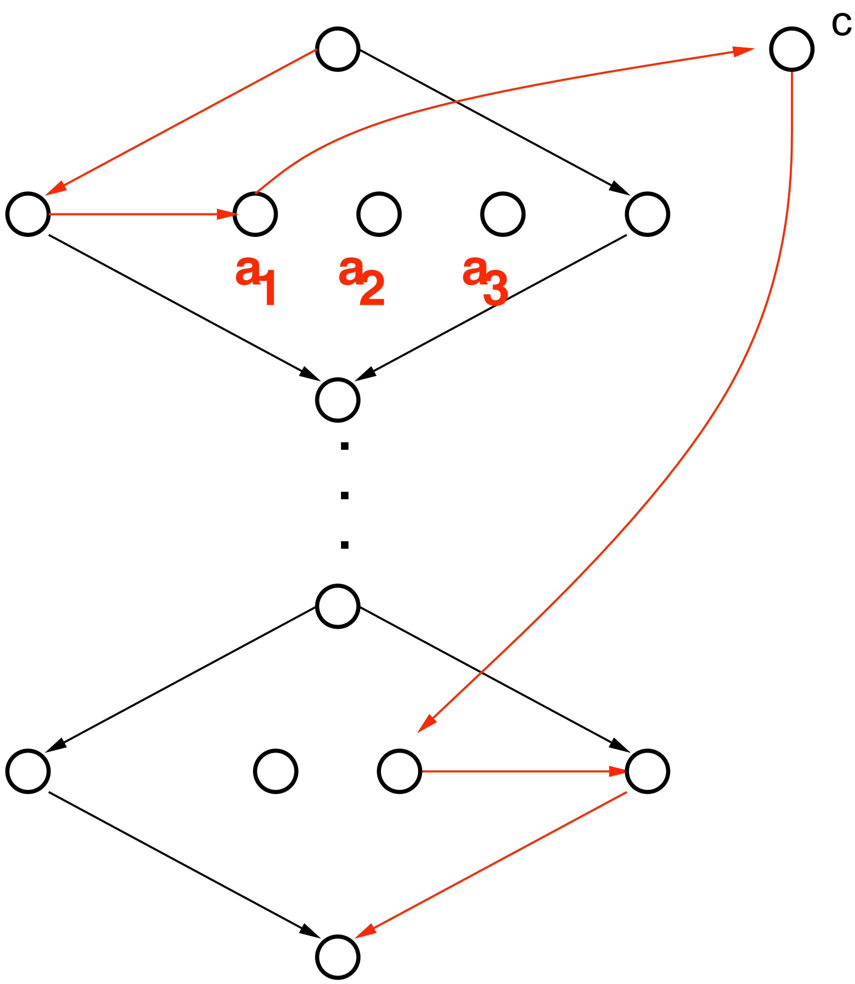
- if x_i diamond zig-zags, assign **true**.
- if x_i diamond zags-zig, assign **false**.
- each clause vertex appears once
- source of detour determines which literal is assigned **true**.

Hamiltonian Path

Claim: Every hamiltonian path in G is normal.



Hamiltonian Path



- only arrows to a_2 from a_1, a_3, c
- paths from a_1 or c go elsewhere
- path from a'_3 would leave no exit

Any hamiltonian path is normal, Q.E.D.